Teachers Repositioning Culturally Diverse Students as Doers and Thinkers of Mathematics

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Interaction and dialogue are seen as essential components of mathematics classrooms of the 21st century. In this paper we explore the pedagogical actions a teacher takes to reposition his diverse learners as active and engaged participants in the classroom. The findings illustrate the need for explicit teacher modelling of ways for students to participate and explain and justify reasoning. We illustrate how teacher actions led to agentic students and a shift from social to sociomathematical norms in the construction of mathematical explanations, justification and generalisations.

In recent times there has been increased focus on teaching and learning of mathematics within classrooms. In these classrooms the students are positioned as problem solvers and required to have effective communication and interaction skills. Classrooms are no longer conceptualised as teacher led; the vision is of teachers and students actively working together to enhance mathematical understanding through effective mathematical practices. Such classrooms place student reasoning through explanation, justification and validation at the centre of classroom activity (Hunter, 2008). Classrooms demonstrating such characteristics are termed inquiry mathematics classrooms (Bell & Pape, 2012; Goos, 2004). Although the research is considerable on inquiry classrooms, given the complexity of the interaction patterns used in them, teachers still struggle to implement them (Hunter, 2008). We recognise that teachers need many models of the implementation of the participation patterns used in inquiry classrooms. This paper will provide another view of how a teacher repositioned his culturally diverse students to be doers and thinkers of mathematics. The specific research questions explored in this paper are:

- What pedagogical actions did the teacher use to reposition the students?
- How did changed patterns of interaction support or limit individual opportunities for mathematical learning?

This paper uses a sociocultural framework. Within this perspective the teaching and learning process is viewed as both academic and social (Pirie & Kieren, 1994; Voigt, 1994). Using this lens supports focusing on teacher actions and student responses in order to understand how pedagogical actions include both social and mathematical structuring.

Literature Review

In inquiry classrooms the teacher and the students work together and use dialogue as a means of communicating what they know in order to make meaning of new ideas or concepts (Rogoff, 1995). The teacher and students jointly participate in activities and learning which allow them to develop a mastery of skills and potential for further learning (Bell & Pape, 2012; Rogoff, 1995). Learning is a process of interthinking (Mercer, 2000).

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Together, teachers and students engage in collective dialogue and activity to create a mutual space known as the intermental development zone. Within inquiry classrooms the structures are reorganised so that students participate in learning by active engagement in doing and talking mathematics (Askew, 2012). Lampert (2001) contends that as students engage in interactive dialogue they have opportunities to critically consider what is presented through questioning and evaluative feedback. In inquiry classrooms equal emphasis is placed on student induction into mathematical practices and understanding of mathematics.

Teachers take an important role in the development of an inquiry environment. Many researchers (e.g., Goos, 2004; Hunter, 2010; McClain & Cobb, 2001; Wood, Cobb, & Yackel, 1995) illustrate the changed role of teachers in inquiry classrooms. The teacher's role is to facilitate the talk and expand conceptual knowledge so that learning becomes a shared realisation. For example, Goos (2004) illustrated that it was the teacher actions which facilitated the creation of a mathematical inquiry classroom. In the first instance, the teacher engaged in deliberate acts of teaching, including modelling processes, structuring social interactions, and linking concepts to mathematical language and symbols. The teacher also created the expectations that students would explain solutions to others and learn through collaborative activity.

In order for students to learn how to participate and contribute effectively in such classrooms, they have to learn and enact specific social and sociomathematical norms. The development of these norms requires the teacher to reposition himself/herself as part of the learning community, as opposed to being the authority figure in the classroom. Classroom norms shape student participation. Through establishing acceptable social norms, students develop social autonomy in mathematics and are accountable for their behaviour. Wood and her colleagues (1995) reported how a teacher repositioned herself from monitoring and supervising students to being part of the interactions with the students as they completed mathematical tasks in small groups. The teacher intervened at intervals in each group and fostered cooperation and mutual exchanging of ideas. By instilling a belief and value in social cooperation, students were able to make sense of each other's explanations and justifications and become mutually supportive.

In contrast to the social norms which regulate social engagement sociomathematical norms are explicit to mathematical activities. They include evaluating mathematical concepts which underpin different strategies and utilising mathematical arguments to reach agreement (Hunter, 2010; McClain & Cobb, 2001). Holding students accountable for their explanations and justifications plays an important role in them developing effective sociomathematical norms. McClain and Cobb (2001) investigated the actions of a teacher in creating sociomathematical norms in a classroom with students aged 6 years. The teacher proactively enhanced development of sociomathematical norms by directing and intervening in students' interactions. The students were held accountable for making acceptable mathematical explanations and justifications. Explanations and justifications had to be acted out on mathematical objects in order to be acknowledged. As a result, the students made significant progress in learning mathematics with understanding through making mathematically sound judgments. Through development of explicit mathematical beliefs and values (for example, being able to explain and justify mathematical assertions) teachers empower students to enhance their autonomy and identity in the mathematics classroom (Cobb, 2000).

Methodology

The data presented in this paper reports on one classroom at an urban primary school in Auckland, New Zealand. The participants were aged 9-10 years and came from middle to low socio-economic home environments. Multiple ethnicities were represented. The teacher had one year of teaching experience. He was selected because he used an inquiry approach in his mathematics classroom. The study was conducted over 4 months. Data were collected in March as the teacher began to establish the norms for a community of inquiry. Data were collected again in July after sixteen weeks in order to compare and contrast changes in the enacted social and sociomathematical norms in the classroom.

Multiple forms of data were collected. These included a questionnaire, a Likert attitude scale, video recorded observations of classroom lessons, video-stimulated recall interviews, reflective discussions with the teacher, field notes and classroom artefacts. The data were triangulated in order to verify findings and to ensure the validity of the project. Analysis of the data consisted of comparing and contrasting responses from both phases of the data collection. Emerging themes and patterns were determined and analysed drawing on themes used by previous researchers.

The format of each of the four lessons in March and in July was the same. The lessons were 50-60 minutes in length. The teacher began each lesson with the presentation of a contextual mathematical problem. The students were then divided into smaller peer groups of three-four participants. The groups worked to solve the problem collaboratively for approximately 15-20 minutes. As they worked the teacher monitored the mathematical activity and facilitated group discussions as necessary; furthering dialogue and development of social and sociomathematical norms. This was followed by a large group sharing and a teacher facilitated discussion of problem solutions.

Findings and Discussion

Setting up the Social and Sociomathematical Norms for an Inquiry Classroom

The teacher placed an immediate focus on reorganising classroom structures so that all members were actively engaged in collaborative learning in an inquiry classroom. For example in interview he stated:

> Teacher: It is vital that students are given clear expectations on how they are to act individually and collaboratively within the classroom. Together, the students and I promoted our learning space as a safe environment where everyone is encouraged to take risks in their thinking and verbalise their thoughts. We establish the expectation that we must support each other and work together to solve problems.

His statement illustrates that he is aware that this community is in the process of being structured. His following statement shows his awareness that constructing a learning community is a lengthy process:

> Teacher: By supporting them to ask questions and seek clarification, their confidence and understanding will increase. The students must believe that they can do it.

The pedagogical actions he took during the lessons focused on clear expectations that during mathematical activity students were required to be active members of the learning community. For example, before a group presented a mathematical explanation to the larger group he ensured that all listeners understood their role:

> Teacher [gesturing to all the students]: John, you are going to start explaining, but before you start, what is your responsibility as citizens of this community?

Students [in unison]: Pay attention.

Recognising that at this early point in structuring the norms of the learning community the students needed explicit models of what paying attention meant he pressed further:

Teacher: How is John going to know that you are paying attention? What are you going to be doing?

Chris: Looking at him.

Teacher: Looking at him. Right, all of you move in a little closer.

Through his revoicing he indicated that he wanted close listening. When John starts to explain his voice is soft. The teacher asks the listeners if they can hear and they say no:

Teacher: So what do you need to do?

Students: Make him speak louder.

David [Taking the initiative]: Can you speak louder, John?

The teacher's action showed that explaining students needed to speak loudly. In turn, he emphasised that the listeners were required to actively engage and if they had difficulties it was their responsibility to state that the explainer needed to raise his voice.

Responsibility to sense-make was also one of his key foci in the early stages of constructing the learning community. For example, he observed a student who had a puzzled look while listening to the explanation:

Teacher: Pamela, do you understand what John is talking about?

Pamela: No.

Teacher: So what should you do if you don't understand?

In this way he reinforced the responsibility of the students to monitor their understandings. When Pamela looks blankly at him he asks the whole group:

Teacher: Who knows what we should do?

Lavinia: Like, tell them in a different way.

He revoices to clarify what the students need to do:

Teacher: You mean, ask them to explain it in a different way?

Throughout the discussion he attended carefully to how the students were engaging in the mathematical explanations. If he noted a lack of attention, or a confused look, he engaged the students with opportunities to alter their behaviour by asking open-ended questions, rather than telling what to do or how to do it. As a result, he positioned them as learners responsible for their sense-making of the proffered explanations. These actions indicated their need to become more agentic in their meaning making.

The way in which the students engaged in small group activity was also attended to. The social norms for how groups worked together were repeatedly addressed as illustrated in the following lesson excerpt:

Teacher: So what is your responsibility when you work together to solve this problem?

Xavier: We need to work out the answer to the question.

Noting the focus on the answer rather than constructing a reasoned explanation he presses further to make the students provide a model of the required behaviour:

Teacher: How are you going to do that?

Sue: I think we have to write it down so everyone in our group can see our answer

To ensure that all members engage in the reasoning he probes deeper:

Teacher: Do you think you have to do something before that?

Luke: We have to talk about the question.

Teacher: What do you mean by that?

Luke: Well, like make sure we understand the question.

Teacher: Yes, you have to think together about what the problem is asking you to do; you have to understand what it is asking before you try to solve it. What happens when you have all understood what the problem means?

The teacher has provided multiple ways for the students to understand that they must all make sense of the problem before constructing an explanation through his revoicing. He then explores further to ascertain if they understand how to develop an explanation. The students state:

Robert: We have to try and figure out the answer. I know we have to work with our buddies in our groups, so not on our own.

May-Lin: Yes, we have to talk to each other to find out what everyone is thinking about the answer.

Teacher: Can anyone add more about how this works? No? Ok, let me remind you that everyone in the group needs to work through a solution step-by-step making sure you understand every part of it. It is important to find a group explanation. What do I mean by that?

David: Everyone in the group must understand every step to get to the answer. Anyone in our group must be able to explain properly so that everyone can understand.

David's response indicates that he is beginning to understand the expected norms for constructing a group explanation. The teacher also took the opportunity to model both how to construct and provide an acceptable mathematical explanation:

Teacher: Now, Anthony, I would like your group to share how you solved the problem. I would like you to explain carefully how you got the answer. You need to show us your thinking step-by-step as you explain to us all how your got the answer.

Anthony hesitates indicating that he is not confident:

Teacher: Anthony, don't worry, we are all taking a risk. What is important is that you try and explain your answer so we can hear your thinking and all of us can understand how your group was thinking when you worked the answer out. Just try your best.

Anthony: First, we had to work out how many lollies Sarah had at the beginning...

The teacher's actions press the students to shift beyond the social norm of making explanations to more closely draw on what makes a conceptual explanation as a sociomathematical norm.

Constructing a learning community was also important. The teacher closely watched how the students were assuming individual and group responsibility. For example, when observing that a student focused solely on explaining to him he directed her to look and explain her explanation to everyone:

Teacher: Angela, when you are explaining, I would like you to look around at everyone's face and see if everyone is listening, or if someone has a question. What we are doing is bringing the control of the lesson to the person who is explaining. See if you have everyone's attention, look for people with their hands up, or even better listen for when someone says "excuse me, Angela". Everyone in this larger group has a responsibility to listen and speak up so that we know that everyone understands each explanation.

He noted also the passivity of the listeners and realised that the students had placed responsibility on him to sense-make rather than assuming a more active role themselves:

Teacher: Ok, so now we need to start getting some real mathematical talk going on. Angela, where did you get 177 from? Where did you get 25 from? Why have you added them together? These are examples of the sorts of questions you all need to ask Angela so that she can explain mathematically how her group solved the problem. Angela, please continue.

He has modelled questions the students need to ask so that the explanations become more conceptual. He presses further and in doing so reinforces the sociomathematical norm of what makes a clear mathematical explanation.

> Teacher: Angela, why did you think it was easier to add 25 to 100, rather than adding 25 to 177? How do you make this explanation mathematical, what is the maths involved?

The classroom structures were beginning to be reorganised to allow development of effective mathematics learning within a learning community.

Maintaining the Social and Sociomathematical Norms for an Inquiry Classroom

In the second phase of the study, there was clear evidence of how the mathematical authority was shared more equally. A learning environment which recognised the relationship between thinking and learning had been established through the reorganisation of the classroom structures. Most students questioned and probed the explanations until justification and proof were provided. The teacher explained:

> Teacher: I believe that collaborative learning can only work where the children feel safe and confident to express themselves fully. Only after this safe environment has been established will you witness the whole class expressing themselves. It is also paramount that the teacher learns to step back and allows the children to work out a problem or discuss something without interruption. Over time these students' confidence has grown and when they work collaboratively they are able to recognise lost focus and actively engage others through questioning.

The teacher has emphasised that a collaborative learning environment needs to be nurtured. His belief in the importance of evenly distributing mathematical authority in the classroom is evident in the way he described his own repositioning.

A clear shift in focus was evident in the lesson observations. The teacher now encouraged persistence and perseverance in pursuing mathematical arguments. He placed an emphasis in their responsibility to engage in argumentation both in interactions and in a way that promoted metacognitive reasoning:

> Teacher: If you think you have an answer, prove it. Ask yourself questions.

Individual interview responses supported observations that students acknowledged the importance of explanation and justification. For example, when a student was shown a video-record and asked what was happening he stated:

> Luke: Well, Pamela said that she had worked the answer out in her head, but I was confused because I didn't know the answer.

Luke emphasises that Pamela is obligated to provide proof for her solution strategy.

> Luke: So we told Pamela to tell us how she got the answer. She started telling us, but I didn't understand, so I knew I had to ask her to explain it differently, otherwise I wouldn't get it. I asked her to prove how she got the answer.
>
> Researcher: And did she prove it?
>
> Luke: Yes, because after she proved it, I got it, I could see how she got there.

The students knew that they were expected to provide acceptable explanations and justification but also it was their responsibility to question until they had full understanding.

Students had become more agentic in sense-making and they were beginning to develop generalisations within the sociomathematical norms enacted in the classrooms. Many students realised that sense-making encompassed not only understanding others' explanations but analysing the differences and similarities across them. For example, the following episode describes how students furthered their mathematical understanding through negotiating the meaning of mathematical difference as they shared their solution strategies:

> Alice: We added $100 and $100 together and that gave us $200. Then we added $50 and $40 and that gave us $90. Then we added $3 and $7 so that gave us $10. Then we added all these amounts together and got $300.
>
> Teacher: Did anyone solve this problem in a different way?
>
> Tony: Our group said split $153 into $100, $50, and $3. Then we split $147 into $100, $40, and $3. Then we added the $100 together and then we went $40 + $50 is $90.

Sonja challenges Tony's explanation. She describes how it is identical to Alice's strategy and they have both used the associative property. She asserts that her group's strategy is different and explains why:

> Sonja (interjecting): But that is the same as Alice, you have just swapped the groups around. I think our one is actually different.

Sonja then explains how her group used a more sophisticated strategy in using compensation through recognising the relationship of $157 and $143 by doubling $150 to get $300.

The students knew they had the authority to compare the similarities and differences between their group's solution strategy and others.

Conclusions

This paper has outlined the pedagogical actions the teacher took to establish a community of learners. The teacher was central to the changes enacted. Many of the pedagogical actions he facilitated match with those other researchers have shown to be important. For example, Goos (2004) illustrated the importance of teachers focusing on the development of students' cognitive and metacognitive strategies by asking significant mathematical questions, enabling collaboration, and holding students accountable for each other's learning. As both Goos (2004) and Hunter (2008; 2010) previously show he recognised the importance of explicitly enforcing an approach to learning mathematics that encompassed responsibility to others and this resulted in students taking ownership of their learning.

Initially the teacher took time to establish an inquiry classroom. It was a lengthy process developing the classroom norms and expectations and required explicit modelling of the processes and interactions. This parallels the findings of Wood and her colleagues (1995). As they illustrated, the expectation that students collaborate supported the students to successfully negotiate meaning but this took time. Likewise, in this study, it was evident in the latter stage of the study that the expectations had become embedded. The students took responsibility for their reasoning and the reasoning of others during mathematics lessons. Through their active engagement in discussions the students were granted affordances which increased their agency.

The teacher had repositioned himself from being the sole authority in the classroom. As Goos (2004) and Wood et al., (1995) show, his realignment as a member of the learning community now supported active student participation and accountable. Furthermore, in line with the research of Hunter (2010) and McClain and Cobb (2001) the students worked within what the learning community had as a shared expectation of what made an acceptable mathematical explanation, justification and generalisation. The students had learnt to use dialogue as an effective tool to communicate their reasoning. As Rogoff (1995) and Mercer (2000) suggest, this supported them to develop thinking spaces and make meanings which progressed their conceptual understanding of mathematics. The joint participation opened up their potential for further learning.

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